Level Structure of ^{10}C

V. Z. Goldberg and G. V. Rogachev[1]
[1]*Department of Physics, Florida State University, Tallahassee, Florida 32306*

In this work (more details can be found in [1]), we are attempting spin-parity assignments for the excited levels in ^{10}C below7 MeV using the data [2, 3] and new results for the $T = 1$ α-cluster states in ^{10}B [4]. Isotope invariance allows us to relate the data available for ^{10}Be and ^{10}B with the ^{10}C spectrum. We apply the Coulomb displacement energies and most importantly, widths and decay modes of the resonances to suggest the spin-parity assignments.

I. POTENTIAL MODEL

The conventional Woods-Saxon potential was used to evaluate the Coulomb displacement energies that are sensitive to the orbital angular momentum, binding energy, and structure of the states and also the single-particle widths of the states. The depth of the well was adjusted for each state to fit thebinding energy in ^{10}Be. A radius of $R = 1.25 \times 9^{1/3}$ fm and a diffuseness of $a = 0.65$ fm were used for the centralpart of the potential. For the spin-orbit potential we use Vso = 6.4 MeV, Rso = $1.3 \times 9^{1/3}$, and $a$so = 0.64 fm. The Coulomb potential was that of the homogeneously charged sphere of $Rc = 1.17 \times 9^{1/3}$. The parameters of the single particle potential fit the ground-state (g.s.) binding energies for mirror pairs ^{9}Be-^{9}B and ^{13}C-^{13}N. The $2s1/2$ and $1d5/2$ single-particle states were also included into the fit for the ^{13}C-^{13}N pair. We obtained agreement for the states included in the fit to within 100 keV. The Coulomb shifts of α-cluster states (analogs of 0^{+}_{2} and 2^{+}_{3} in the ^{10}Be spectrum) were calculated using the Woods-Saxon potential from Ref. [3] with $V = -119$ MeV, radius and charge radius of 2.58 fm and 2.27 fm respectively, and diffuseness of $a = 0.677$ fm. Such a potential generates the correct binding energy for the 0^{+}_{2} level in ^{10}Be and the correct excitation energy of 2^{+}_{3} , and also produces deeply bound "forbidden" states to account forthe Pauli principle. Note that the results of potential model calculations are not very sensitive to the specific choice of potential parameters as long as the excitation energies of the states are reproduced.

II. ANALYSIS

The analysis starts with fitting the well depth of the potential to reproduce the binding energy of the states in ^{10}Be. Then the excitation energy for the corresponding isobaric analog state in ^{10}B is calculated. For this, we use the average between the energy relative to the threshold for ^{10}B decay into n + ^{9}B (the same as the energy relative to the ^{10}Be decay into ^{9}Be + n) and the energy relative to the threshold for ^{10}B decay into ^{9}Be + p (calculated with the potential found for ^{10}Be and exchanging the neutron for the proton). The single-particle nucleon (or α for the 0^{+}_{2} and 2^{+}_{3} levels) widths Γsp are determined from the potential model. The width of the resonance is defined using the behavior of the wave function in the interior as in Ref. [5]. The known widths of the $T = 1$ resonances in ^{10}B (mainly ^{9}Be + p) [13] are proportional to the spectroscopic factors (SFs). We used the expression $C^2S = \Gamma\text{exp}/\Gamma\text{sp}$, where C is an isospin Clebsch-Gordan coefficient ($C^2 = 1/2$ for the ^{9}Be + p decay of ^{10}B). The ratios of the experimental

proton widths (when known) of the resonances to the calculated single particle widths were considered as the spectroscopic factors [^{9}Be(g.s) + n for ^{10}Be or ^{9}B(g.s) + p for ^{10}C]. Reduced widths for proton and neutron decay should be the same for the T = 1 states in ^{10}B if isospin is conserved. When the known partial proton widths in ^{10}B were used to obtain the SFs, the corresponding Γexp/Γsp ratio is multiplied by a factor of 2 to get a SF in ^{10}Be and ^{10}C. These values were considered as experimental values and summarized in the seventh column of Table I. The SFs from the ^{9}Be(d,p) reactions [6, 7] are given in the fourth column of Table I. We then calculated theoretical SFs in the framework of the shell model (SM) using the code COSMO [8] (column 3 of Table I). The *psd* valence space with WBP interaction [14] was used. 0–2 $\hbar\omega$ excitations were considered for the positive parity states and 1–3 $\hbar\omega$ for the negative parity state. As it is clear from Table I, generally there is reasonable agreement between the experimental and

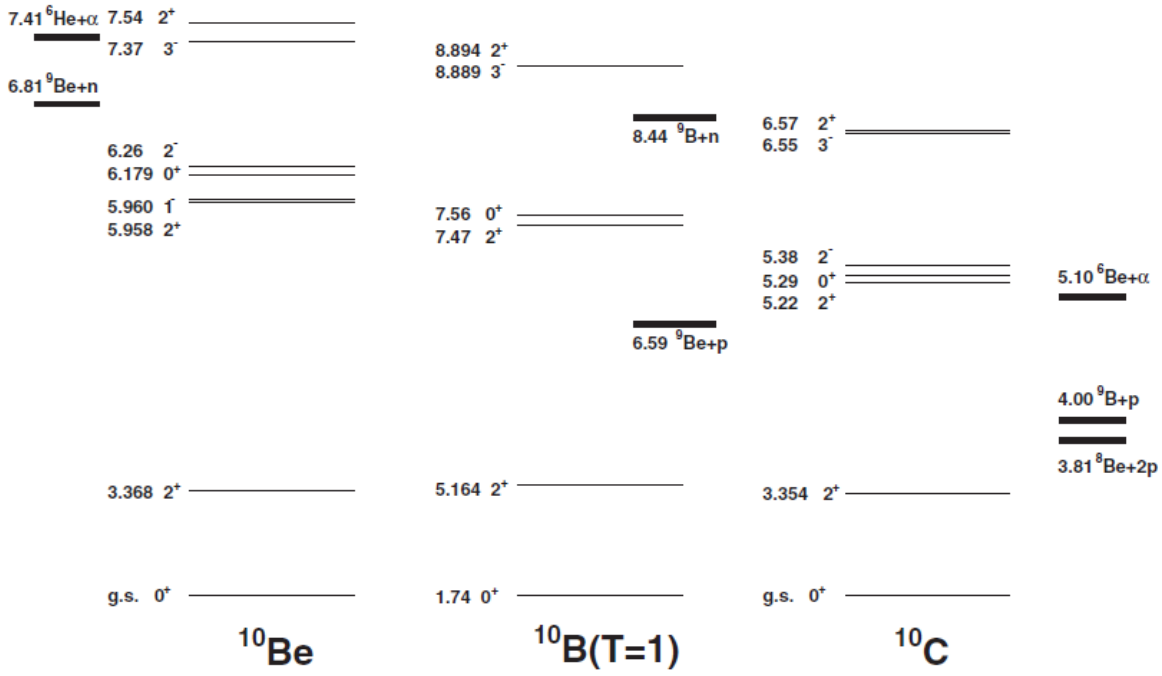

FIG. 1. Levels diagram for the $A = 10$, $T = 1$ isobaric multiplet.

calculated SFs. We used the SM predictions to calculate the Coulomb displacement energies of these negative parity states in ^{10}C, but we realize that the uncertainty of these calculations is much larger than for the other states. Fortunately, the width of the 2−1 state in ^{10}C is determined by the l= 0 SF and is not affected by this discrepancy. As for the width of the 1− state we present the lower limit. It is worthwhile to note the remarkable stability of the differences in excitation energies for the cluster 0+ and 2+ levels calculated in ^{10}Be and ^{10}C. This equidistance is quite different from what should be expected for single-particle nucleon resonances with l= 0 and l = 2. The well known Thomas-Ehrman effect [9, 10] shifts down the l= 0 unbound single-particle levels in mirror proton-rich nuclei. It is different for the α-particle resonances mainly due to larger reduced mass decreasing the role of the orbital momenta. This behavior can be considered as a specific characteristic of the cluster states.

TABLE I. $T = 1$ excited states above 5 MeV in $A = 10$ nuclei.

							^{10}Be	^{10}B($T = 1$)	^{10}C				
J^π	ℓ_N	S_{th}[a]	$C^2S_{exp}(^{10}Be)$[b]	$\Gamma_p(^{10}B)$ (keV)	$\Gamma_{sp}(^{10}B)$ (keV)	$2\times\frac{\Gamma_p}{\Gamma_{sp}}$[c]	E_{exp} (MeV)	E_{exp} (E_{calc}) (MeV)	E_{calc} (MeV)	Γ_{calc}[g] (keV)	E_{exp} (MeV)	Γ_{exp} [16] (keV)	Γ_{exp} [20] (keV)
2_2^+	1	0.73	0.54	65(10)	200	0.65(10)	5.958	7.47 (7.50)	5.16	210(30)/170	5.22	294(16)	225(45)
1_1^-	0	0.40		100(10)	1240	0.16(2)[d]	5.960	7.43 (7.49)	5.1[e]	>180			
	2	0.15											
0_2^+	1	0.07		2.65(18)	250	0.021(2)	6.179	7.56 (7.58)	5.39	10[h]	5.287	106(11)	
2_1^-	0	0.11	0.132	210(60)	2700	0.15(4)	6.263	7.74 (7.79)	5.4[e]	370(100)	5.38 [21]		300(60)
	2	0.53	0.065										
3_1^-	2	0.57	0.53	75(10)[f]	350	0.43(6)	7.371	8.89 (8.93)	6.70	140(20)/175	6.553	214(31)	
2_3^+	1	0.03	0.007	7(2)[f]	2200	0.006(2)	7.542	8.89 (8.87)	6.70	90[h]	6.568	172(31)	190(35)

[a]Theoretical spectroscopic factor S_{th} for the $N\otimes 3/2^-_{g.s.}$ configuration. The square of the isospin Clebsch-Gordan coefficient, which is unity for ^{10}Be and ^{10}C, and 1/2 for $T = 1$ states in ^{10}B, is omitted.
[b]SF from ^{9}Be(d, p) experiments [22,23].
[c]This column gives the SF for the corresponding state determined from the ratio of the known proton partial width of the $T = 1$ states in ^{10}B to a single-particle width calculated with the potential model. The ratio is then multiplied by a factor of 2 to account for the isospin Clebsch-Gordan coefficient.
[d]The 1^- at 7.43 MeV in ^{10}B has mixed isospin, and the experimental spectroscopic factor determined from the width of this state may be unreliable. See text for additional comments.
[e]Assuming an experimental $\ell = 0$ SF and a theoretical $\ell = 2$ SF, we estimate uncertainty of $\pm$200 keV for this value due to uncertainties in the SF.
[f]The proton partial width in ^{10}B that was used to determine the SF is from Ref. [18].
[g]The widths for all states except for the cluster 0_2^+ and 2_3^+ were calculated as a product of the spectroscopic factor given in column 7 and the single-particle width for the $p + ^9$B system. If the SF from the (d, p) reaction (column 4) is outside of the uncertainty given in column 7, then the second width that corresponds to the SF from (d, p) is also shown (after the slash).
[h]The width for this state was calculated as a sum of the α single-particle width from the $\alpha + ^6$Be potential model and the partial width for the proton decay to the ^{9}B(g.s.).

It is seen (Table I, column 11) that the α-cluster states may be relatively narrow resonances in ^{10}C. Thus, the 100 keV resonance at 5.29 MeV [2] can only be the 0^+_2 state, and the 170 keV resonance at 6.6 MeV is likely the 2^+_3 cluster state. The calculated widths of the cluster states, 0^+_2 and 2^+_3, are too small, however, if only the cluster decay (with $S\alpha = 1$) and the proton decay to the ground state in ^{9}B are taken into account. Several charged-particle decays are energetically possible for the ^{10}C excited states, while the mirror decays are not possible in ^{10}Be. The account of the decays to the excited states in ^{9}B for all states (except for the 0^+_2 and 2^+_3) results in a 10–15% increase of the widths shown in Table I. As for the cluster 0^+_2 and 2^+_3 states, the lowest $2p$ decay is a new and important channel. It is the only channel which can provide for the increase of the width of the 0^+_2 state. A simplified consideration of the $2p$ decay as a di-proton decay in the potential model shows that, if the spectroscopic factor for this decay is about 0.15, then it provides for 100 keV of the total width. In this case the $2p$ decay will be dominant, in agreement with the experimental observation in Ref. [2]. A similar consideration for the 2^+_3 cluster state would result in the increase of its width by ~100 keV, also improving the agreement with the experimental data. Now our interpretation of the results of Ref. [2] is the following.

A. States near 6.6 MeV in ^{10}C

The group at 6.6 MeV excitation energy in ^{10}C consists of two nearly degenerated levels: 3− at 6.55 MeV and 2+ at 6.57 MeV. (Here and below we are using experimental excitation energies given in column 12 of Table I, which are known with a precision of 50 keV [2].) The major mode of decay for the 2+ level is into α + ^{6}Be (this state was found to be an extreme α-cluster state in Ref. [4]). The analog 3−

state in ^{10}B has relatively large reduced α +^{6}Li(0+; T = 1) width as well ($SF\alpha$ = 0.42 [3]).We expect a much stronger population of the 3− state than the 2+ state in the inelastic scattering [2] because of the collective enhancement of L = 3 transitions in light nuclei in this energy region [11]. Therefore, we suppose that the authors of Ref. [2] observed the structure at 6.56MeV, which looked like a broad level decaying to α +^{6}Be with a width of <370 keV that is due to unresolved 3^-_1 and 2^+_3 . However, the dominant mode of the 3− level decay is to ^{9}B +p. This decay results in observation of the strong population of the level at 6.553 MeV with a width of 214(31) keV [2] (which is the real width of the 3− level because the 9B + p partial width of the 2+ level is negligible). We estimate that the admixture of the 2p+^{8}Be decay for the 2^+_3 level is ~1/2 of the total width. This results in observation of the narrow structure of 172 keV (the width of the 2+ level) in the 2p + ^{8}Be channel, reported in Ref. [2]. The energy of the cluster 0^+_2 and 2^+_3 levels depends on the presence of the 2p + ^{8}Be configuration. The admixture of this configuration at the level of 15% of the maximum diproton width, which was needed to explain the widths of the 0^+_2 and 2^+_3 resonances, results in a decrease of ~100 keV of the excitation energy of the 0^+_2 and 2^+_3 states, improving the agreement between the calculated and experimental Coulomb displacement energies. The maximum diproton width was calculated using the 2p + ^{8}Be potential model. As for the 3− state, the calculated excitation energy in ^{10}C should be corrected for the admixtures of the cluster and the collective configurations.

B. States near 5.3 MeV in ^{10}C

The narrowest resonance in the 5.2–5.3MeV group is 0+ at 5.29 MeV. The dominant decay mode for this state is 2p + ^{8}Be because of small penetrability for the α + ^{6}Be channel. (A similar decay is observed for the $2+_3$ level at 6.6 MeV). All other resonances close to 5.2 MeV should decay into the ^{9}B + p channel. The dominant population of this structure can be explained if 2^+_2 and 1^-_1 contribute to the peak at 5.2 MeV. While the measurements in Ref. [2] presented more detailed information on the ^{10}C states in question than the former experiments, there is an evident difference at 5.38MeV where a peak with width of 300(60) keV was reported in the ^{10}B(^{3}He,^{3}H) reaction [12]. The excitation energy and width of this peak are close to our calculations for the 2− state. The states with abnormal parity can be populated in inelastic scattering experiment of Ref. [2] only due to the second-order effects. Therefore, we suppose that the 2− state was not observed in Ref. [2].

III. SUMMARY

We considered states in the 5–7 MeV excitation energy region in ^{10}C and proposed spin-parity assignments for these states. In particular we showed that the states (0+ and 2+) with the cluster (α + ^{6}Be) structure have the narrowest widths in this excitation region. We apply a rather common procedure of using shell-model wave functions to calculate the Coulomb shifts and widths for the states with evident single-particle spectroscopic factors. A similar procedure with cluster potentials accounting for the configurations forbidden by the Pauli principle was used to explore the isospin invariance for the cluster states. While different cluster potentials are conventional instruments to consider cluster states, it is difficult to find examples of the applications for the mirror nuclei. This is because the experimental data on cluster states in mirror nuclei are very rare. We noticed a remarkable (in comparison with the behavior

of the nucleon single-particle states) equidistance of 0+–2+ cluster state energies in mirror nuclei. Our test calculations showed that more complete data on the unknown members of the cluster band (we expect a 4^+ state at 10.1 MeV with a width of ~600 keV) would provide for important information on the details of the cluster potential, first of all on the number of nodes of the cluster wave function. This number is determined by the specific orbitals occupied by the nucleons in the cluster and has an influence on the moment of inertia of the band. The spin-parity assignments suggested here became possible due to recent experimental data containing information on the different decay modes of the states. As seen in Fig. 1, more decay channels are open for the states in the proton-rich member of the $T = 1$ multiplet, the exotic $2p$ decay being the lowest one. We have shown that the observation of this channel in Ref. [2] appeared to be very useful for the identification of the cluster levels. The $2p$ partial width is much larger than the single-particle width for the cluster states. When we began this work we hoped that we would obtain an indication for the need to increase the Coulomb radius to an unusually large value for some (cluster) states. Indeed, the calculated excitation energies for the 0^+ and the 2^+ states are higher than the experimental ones by ~100 keV. A 50% increase of the Coulomb radius would be needed to match the experimental data. However, the results of calculations depend on the proper accounting for the presence of the $2p$ channel and also on the number of nodes of the cluster wave function. Based on our analysis we conclude that the partial widths for the $2p$ decay of the cluster 0^+ and 2^+ states are ~100 keV. It is interesting to see if these can be reproduced by the microscopic many-body calculations. We presented evidence that novel measurements of the properties of the proton rich nuclei could be very useful. Even if the quantum characteristics determination is not directly possible in these experiments, the comprehensive analysis of the properties of the states in the isobaric multiplet can be reliable, and a test of the theoretical approaches can be more complete.

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